Tests for Series Convergence

Test	Conditions to Check	Result of Test
Test for Divergence	$\lim\limits_{n\to\infty} a_n \neq 0$	$\sum_{n=0}^{\infty} a_n$ diverges
Geometric Series	$a_n = ar^n$ for some a, r	If $\lvert r\rvert < 1$, $\sum_{n=0}^{\infty} a_n$ converges to $\dfrac{a}{1-r}$ If $\lvert r\rvert \geq 1$, $\sum_{n=0}^{\infty} a_n$ diverges
Integral Test	f is a continuous, positive, decreasing function on $[b, \infty)$ for some integer b and $a_n = f(n)$	$\sum_{n=b}^{\infty} a_n$ converges if and only if $\int_b^{\infty} f(x)dx$ converges.
p-series	$a_n = \dfrac{1}{n^p}$	If $p > 1$, $\sum_{n=1}^{\infty} a_n$ converges If $p \leq 1$, $\sum_{n=1}^{\infty} a_n$ diverges
Direct Comparison Test	a_n and b_n are positive	If $a_n \leq b_n$ and $\sum b_n$ converges, so does $\sum a_n$. If $a_n > b_n$ and $\sum b_n$ diverges, then so does $\sum a_n$.
Limit Comparison Test	a_n and b_n are positive and $\lim_{n\to\infty} \dfrac{a_n}{b_n} = c$ with $0 < c < \infty$	$\sum a_n$ converges if and only if $\sum b_n$ converges
Alternating Series Test	$a_n = (-1)^n b_n$ with $b_n > 0$ b_n is decreasing $\lim\limits_{n\to\infty} b_n = 0$	$\sum a_n$ converges Also, $\lvert s - s_n\rvert \leq b_{n+1}$
Ratio Test		If $\lim\limits_{n\to\infty} \left\lvert \dfrac{a_{n+1}}{a_n} \right\rvert < 1$, $\sum a_n$ converges absolutely If $\lim\limits_{n\to\infty} \left\lvert \dfrac{a_{n+1}}{a_n} \right\rvert > 1$, $\sum a_n$ diverges If $\lim\limits_{n\to\infty} \left\lvert \dfrac{a_{n+1}}{a_n} \right\rvert = 1$, use a different test
Root Test		If $\lim\limits_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} < 1$, $\sum a_n$ converges absolutely If $\lim\limits_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} > 1$, $\sum a_n$ diverges If $\lim\limits_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} = 1$, use a different test